

Fill in the blanks. The correct answer in each case is either “*a scalar*”, “*a vector*” or “*invalid*”.

SCORE: _____ / 4 PTS

[a] $\vec{u} \times (\vec{v} \cdot \vec{w})$ is INVALID ①

[b] $(\vec{u} \cdot \vec{v})\vec{w}$ is A VECTOR ①

[c] $(\vec{u} \times \vec{v}) \cdot (\vec{u} \times \vec{w})$ is A SCALAR ①

[d] $\vec{u} \times (\vec{v} \times \vec{w})$ is A VECTOR ①

Fill in the blanks.

SCORE: _____ / 3 PTS

- [a] $\vec{m} \times \vec{m} = \boxed{\vec{0}} \textcircled{1}$. ANSWER MUST BE A VECTOR,
NOT THE NUMBER 0

- [b] If \vec{r} is a vector of magnitude 8, and \vec{n} is a vector of magnitude 6, and the angle between \vec{r} and \vec{n} is $\frac{3\pi}{4}$ radians,

then the magnitude of $\vec{n} \times \vec{r} = \boxed{24\sqrt{2}} \textcircled{1}$. $\|\vec{n}\| \|\vec{r}\| \sin \theta = 6 \cdot 8 \cdot \sin \frac{3\pi}{4}$

- [c] If $\vec{w} \times \vec{v} = \langle -4, 1, 5 \rangle$, then $\vec{v} \times \vec{w} = \boxed{\langle 4, -1, -5 \rangle} \textcircled{1} - (\vec{w} \times \vec{v})$

Fill in the blanks.

SCORE: ____ / 6 PTS

NOTE: For each part (ie. [a], [b], [c]), you must fill in all blanks correctly to receive any credit.

[a] If line ℓ is parallel to plane ϕ , then the DIRECTION vector of line ℓ and

the NORMAL vector of plane ϕ are PERPENDICULAR.

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[b] If plane ϕ_1 is parallel to plane ϕ_2 , then the NORMAL vector of plane ϕ_1 and

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the NORMAL vector of plane ϕ_2 are PARALLEL.

[c] If line ℓ is perpendicular to plane ϕ , then the DIRECTION vector of line ℓ and

the NORMAL vector of plane ϕ are PARALLEL.

Let P be the point $(1, -1, -3)$.

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Let Q be the point $(-1, 1, -2)$.

Let R be the point such that $\overrightarrow{PR} = 6\vec{i} + 3\vec{k}$.

- [a] Find the area of triangle PQR .

$$\overrightarrow{PQ} = \langle -2, 2, 1 \rangle \quad (1)$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 1 \\ 6 & 0 & 3 \end{vmatrix} \begin{matrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{matrix} = 6\vec{i} + 6\vec{j} - (-6\vec{j} + 12\vec{k}) = \langle 6, 12, -12 \rangle \quad (3)$$

SIMPLY CHECK:

$$(6, 12, -12) \cdot \langle -2, 2, 1 \rangle = -12 + 24 - 12 = 0$$

$$(6, 12, -12) \cdot \langle 6, 0, 3 \rangle = 36 + 0 - 36 = 0$$

$$\text{AREA} = \frac{1}{2} \|\langle 6, 12, -12 \rangle\| \quad (1)$$

$$= \frac{1}{2} \cdot 6 \|\langle 1, 2, -2 \rangle\|$$

$$= \frac{1}{2} \cdot 6 \cdot 3 = 9$$

- [b] Find symmetric equations for the line which is perpendicular to the plane $2y - 3z = 6$, and also contains Q .

$$\vec{d} \parallel \vec{n} = \langle 0, 2, -3 \rangle$$

$$\text{USE } \vec{d} = \langle 0, 2, -3 \rangle$$

$$(1) \quad (\pm)$$

$$x = -1, \quad \frac{y-1}{2} = \frac{z+2}{-3} \quad \text{OR} \quad -\frac{z+2}{3} \quad (1)$$

- [c] Find the standard (point-normal) equation of the plane which contains P , Q and R .

$$\vec{n} \perp \overrightarrow{PQ}, \overrightarrow{PR} \rightarrow \text{use } \vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 6, 12, -12 \rangle \text{ or } \langle 1, 2, -2 \rangle$$

$$\text{USING } P: \quad (x-1) + 2(y+1) - 2(z+3) = 0 \quad (1)$$

$$\text{USING } Q: \quad (x+1) + 2(y-1) - 2(z+2) = 0$$

- [d] Find parametric equations for the line which is parallel to $\frac{4-x}{3} = \frac{2+y}{7} = 5+z$, and also contains R .

$$\vec{d}_1 \parallel \vec{d}_2 = \langle -3, 7, 1 \rangle$$

$$\text{USE } \vec{d}_1 = \langle -3, 7, 1 \rangle$$

$$\langle 6, 0, 3 \rangle = \langle x-1, y+1, z+3 \rangle$$

$$x-1=6 \quad x=7$$

$$y+1=0 \rightarrow y=-1$$

$$z+3=3 \quad z=0$$

$$R(7, -1, 0) \quad (1)$$

- [e] Find a vector of magnitude 5 perpendicular to both \overrightarrow{PQ} and \overrightarrow{PR} .

$$\frac{5}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|} \overrightarrow{PQ} \times \overrightarrow{PR} = \frac{\frac{5}{2}}{\frac{5}{18}} \langle 6, 12, -12 \rangle = \left\langle \frac{5}{3}, \frac{10}{3}, -\frac{10}{3} \right\rangle \quad (1)$$